

The likelihood (not the log likelihood) of the RV component of our orbit model (assuming one instrument) is

$$\mathcal{L}_{\text{RV}} = \left(\prod_i \frac{1}{\sqrt{2\pi(\sigma_i^2 + j^2)}} \right) \exp \left[-\frac{1}{2} \sum_i \frac{(\text{RV}_i - m_i - z)^2}{\sigma_i^2 + j^2} \right]. \quad (1)$$

where m_i is the modeled RV for epoch i , j is the jitter, and z is the instrumental zero point. For multiple instruments, the likelihood is separable and so the following argument still holds. Suppose that I am at a particular step in the MCMC chain with the model parameters for the orbit and the jitter fixed, i.e., everything except the zero point. If I want to reduce the dimensionality of the problem, I would like to integrate over z at these fixed values of the other parameters. I can do that analytically. First, to make things a little cleaner, I'll define

$$P \equiv \prod_i \frac{1}{\sqrt{2\pi(\sigma_i^2 + j^2)}}. \quad (2)$$

Then

$$\mathcal{L}_{\text{RV}} = \int_{-\infty}^{\infty} dz P \exp \left[-\frac{1}{2} \sum_i \frac{(\text{RV}_i - m_i - z)^2}{\sigma_i^2 + j^2} \right] \quad (3)$$

$$= P \int_{-\infty}^{\infty} dz \exp \left[-\frac{1}{2} \left(z^2 \sum_i \frac{1}{\sigma_i^2 + j^2} - 2z \sum_i \frac{\text{RV}_i - m_i}{\sigma_i^2 + j^2} + \sum_i \frac{(\text{RV}_i - m_i)^2}{\sigma_i^2 + j^2} \right) \right]. \quad (4)$$

Now I will define

$$\tilde{z} \equiv \left(\sum_i \frac{\text{RV}_i - m_i}{\sigma_i^2 + j^2} \right) \left(\sum_i \frac{1}{\sigma_i^2 + j^2} \right)^{-1}. \quad (5)$$

You might notice that this is the same value of z you would get by differentiating χ^2 and setting the derivative equal to zero. In other words, it is the maximum likelihood value of z given the assumed values of the other parameters. With this definition, I have

$$\mathcal{L}_{\text{RV}} = P \int_{-\infty}^{\infty} dz \exp \left[-\frac{1}{2} \left(\sum_i \frac{1}{\sigma_i^2 + j^2} \right) (z - \tilde{z})^2 - \frac{1}{2} \left(\sum_i \frac{(\text{RV}_i - m_i)^2}{\sigma_i^2 + j^2} - \tilde{z}^2 \sum_i \frac{1}{\sigma_i^2 + j^2} \right) \right] \quad (6)$$

$$= P \exp \left[-\frac{1}{2} \left(\sum_i \frac{(\text{RV}_i - m_i)^2}{\sigma_i^2 + j^2} - \tilde{z}^2 \sum_i \frac{1}{\sigma_i^2 + j^2} \right) \right] \int_{-\infty}^{\infty} dz \exp \left[-\frac{1}{2} \left(\sum_i \frac{1}{\sigma_i^2 + j^2} \right) (z - \tilde{z})^2 \right]. \quad (7)$$

The first part has factored out of the integral, and the integral itself is now easy. In fact, with the likelihood written this way, you can see that the probability distribution on z given the other parameter values is a Gaussian centered on \tilde{z} with variance

$$\sigma_z^2 = \left(\sum_i \frac{1}{\sigma_i^2 + j^2} \right)^{-1}. \quad (8)$$

If I do the integral to marginalize it out, I have

$$\mathcal{L}_{\text{RV}} = P \exp \left[-\frac{1}{2} \left(\sum_i \frac{(\text{RV}_i - m_i)^2}{\sigma_i^2 + j^2} - \tilde{z}^2 \sum_i \frac{1}{\sigma_i^2 + j^2} \right) \right] \sqrt{2\pi\sigma_z^2}. \quad (9)$$

A little more manipulation will make this even more intuitive. Recalling my definition of \tilde{z} to make the substitution

$$\tilde{z}^2 \sum_i \frac{1}{\sigma_i^2 + j^2} = \tilde{z} \sum_i \frac{\text{RV}_i - m_i}{\sigma_i^2 + j^2}, \quad (10)$$

I have

$$\mathcal{L}_{\text{RV}} = P \exp \left[-\frac{1}{2} \left(\sum_i \frac{(\text{RV}_i - m_i)^2}{\sigma_i^2 + j^2} - \tilde{z}^2 \sum_i \frac{1}{\sigma_i^2 + j^2} \right) \right] \sqrt{2\pi\sigma_z^2} \quad (11)$$

$$= P \sqrt{2\pi\sigma_z^2} \exp \left[-\frac{1}{2} \left(\sum_i \frac{(\text{RV}_i - m_i)^2}{\sigma_i^2 + j^2} - 2\tilde{z}^2 \sum_i \frac{1}{\sigma_i^2 + j^2} + \tilde{z}^2 \sum_i \frac{1}{\sigma_i^2 + j^2} \right) \right] \quad (12)$$

$$= P \sqrt{2\pi\sigma_z^2} \exp \left[-\frac{1}{2} \left(\sum_i \frac{(\text{RV}_i - m_i)^2}{\sigma_i^2 + j^2} - 2\tilde{z} \sum_i \frac{\text{RV}_i - m_i}{\sigma_i^2 + j^2} + \tilde{z}^2 \sum_i \frac{1}{\sigma_i^2 + j^2} \right) \right] \quad (13)$$

$$= P \sqrt{2\pi\sigma_z^2} \exp \left[-\frac{1}{2} \sum_i \frac{(\text{RV}_i - m_i - \tilde{z})^2}{\sigma_i^2 + j^2} \right]. \quad (14)$$

In other words, marginalizing over z is equivalent to multiplying by $\sqrt{2\pi\sigma_z^2}$ and setting $z = \tilde{z}$ in the likelihood. Even better, for the (trivial) cost of computing \tilde{z} from Equation (5) and σ_z^2 from Equation (8), we have the full probability distribution of z at fixed values of the other parameters. If you want, you can just generate a value of z from this distribution to save as the dummy value in our output MCMC step (though you should use the marginalized likelihood, Equation (14) as the likelihood that MCMC sees), but that is throwing away useful information. Better to just keep the full probability distribution. You could generate arbitrarily many independent z from this distribution to make your chain longer. It would be the same length in the non- z parameters, or you could just do the integral and plot the full distribution in z .